Open-Channel Hydraulics
H.W.#3 Reynolds Equations

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1. Express $\overline{\nabla V^2/2}$ and $\overline{V\times \boldsymbol{curl} V}$ in terms of u, u′, v, v′, w, and w′ for turbulent flows.

2. Express the average ratio of dilatational and shear deformation in terms of mean and fluctuating velocity components for a turbulent flows.

3. Demonstrate that the Reynolds equations can still be written:

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}+\bar{w}\frac{\partial \bar{u}}{\partial z}=\frac{\partial}{\partial x}\left[\frac{p^*}{\rho}+2\nu\frac{\partial \bar{u}}{\partial x}-\overline{u'^2}\right]$$
$$+\frac{\partial}{\partial y}\left[\nu\left(\frac{\partial \bar{u}}{\partial y}+\frac{\partial \bar{v}}{\partial x}\right)-\overline{u'v'}\right]$$
$$+\frac{\partial}{\partial z}\left[\nu\left(\frac{\partial \bar{u}}{\partial z}+\frac{\partial \bar{w}}{\partial x}\right)-\overline{u'w'}\right]$$

and two other equations which will be determined. Indicate the advantage of this form of the Reynolds equations.

4. Write the Reynolds equations in the case of a mean two-dimensional motion. Write the Reynolds equations in the case of isotropic turbulence, i.e., $\overline{u'^2}=\overline{v'^2}$ and $\overline{u'v'}=0$.